15.4 Lecture: Regions in polar coordinates and converting from Cartesian integrals to polar integrals

Jeremiah Southwick
(But really Robert Vandermolen)

Spring 2019

Links

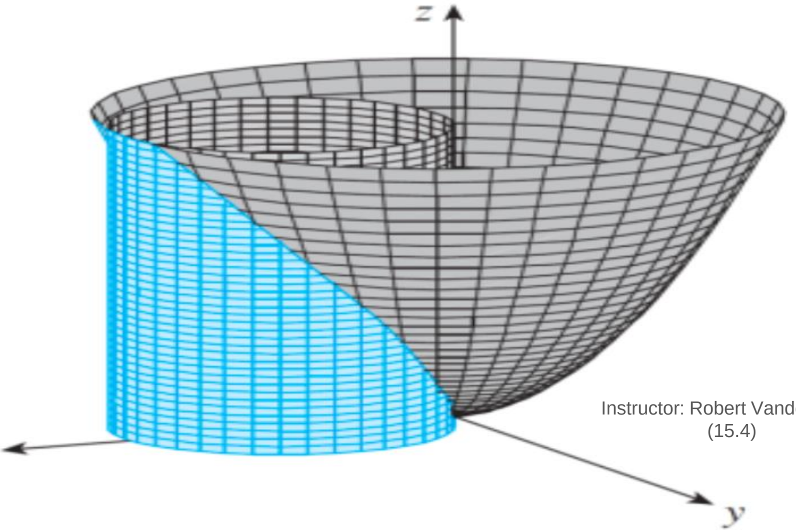
Robert's slides can be found here:

http://people.math.sc.edu/robertv/teaching.html

The 15.4 slides can be found here:

https://docs.google.com/presentation/d/
1-xUgXkCZh0CglmUvGODH8yx6bjv6z4nGlkZLxVPeqZ8

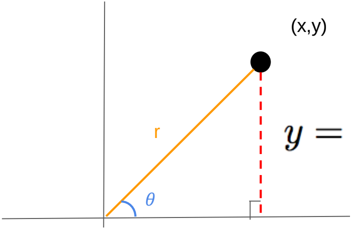
CALCULUS III



Instructor: Robert Vandermolen
(15.4)

Double Integrals with Polar Coordinates!

Let's recall how we relate polar coordinates and rectangular coordinates in the xy-plane...



$$y = r\sin(\theta)$$

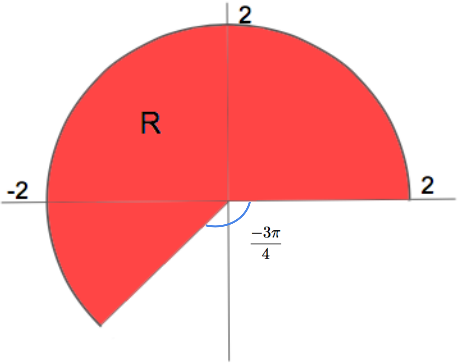
$$x = r\cos(\theta)$$

$$r^2 = x^2 + y^2 \qquad \tan(\theta) = \frac{x}{y}$$

Double Integrals with Polar Coordinates!

Let's practice finding some regions in polar coordinates...

$R:$



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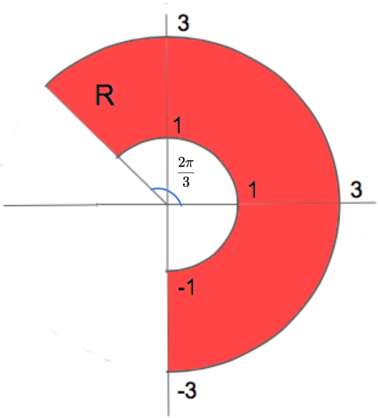
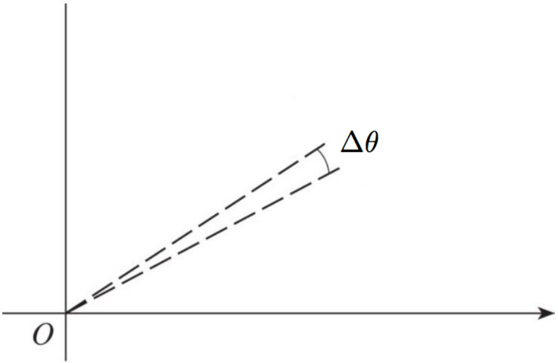


Double Integrals with Polar Coordinates!

In 2-dimensions we can integrate, just as we did befor, this time the areas have a slightly different shape...



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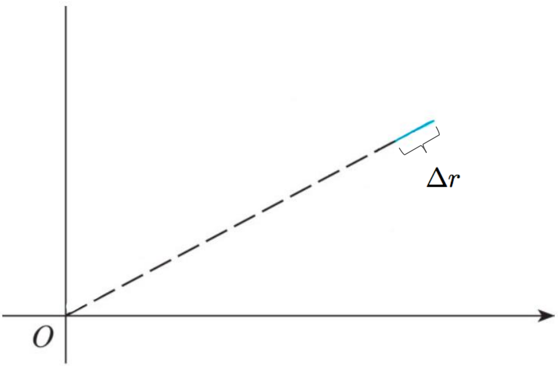
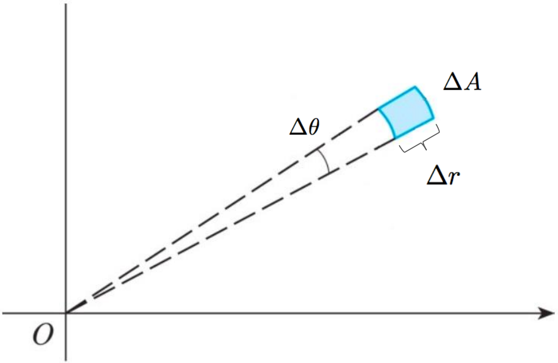


Double Integrals with Polar Coordinates!

In 2-dimensions we can integrate, just as we did befor, this time the areas have a slightly different shape…

We call this region a Polar Sector

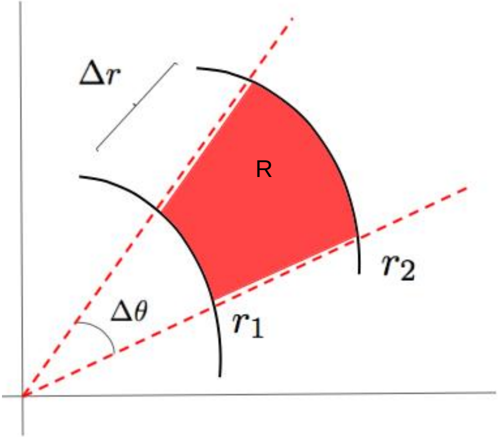


Double Integrals with Polar Coordinates!

Yet we seen how to take a function z=f(x,y), in rectangular coordinates, and set up many interesting double integrals, so to change our integration to polar coordinates, we will need to see how to change dx and dy into dr and dθ…

To do this let's look at the region R, and look at the area

$$\begin{aligned} dxdy = \Delta A &= \frac{(r_2^2 - r_1^2)(\theta_2 - \theta_1)}{2} \\ &= \frac{(r_2 + r_1)}{2}(r_2 - r_1)(\theta_2 - \theta_1) \\ &= \frac{r_2 + r_1}{2}\Delta r \Delta\theta \\ &= r\Delta r\Delta\theta \end{aligned}$$



Double Integrals with Polar Coordinates!

With this we arrive at our change of variable...

CHANGE TO POLAR COORDINATES IN A DOUBLE INTEGRAL If f is continuous on a polar rectangle R given by $0 \leqslant a \leqslant r \leqslant b$, $\alpha \leqslant \theta \leqslant \beta$, where $0 \leqslant \beta - \alpha \leqslant 2\pi$, then

$$\iint_R f(x, y)\, dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

Double Integrals with Polar Coordinates!

Let's practice finding some double integrals with polar coordinates…

Let R be the region between the two circles

$$x^2 + y^2 = 1$$

and

$$x^2 + y^2 = 25$$

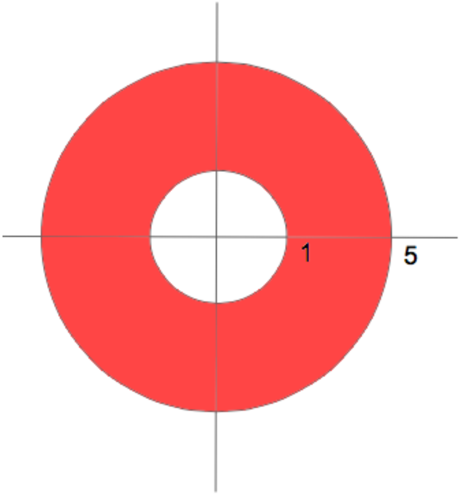
Evaluate the integral:

$$\int\int_R (x^2 + y) \, dA$$

$R:$

$$0 \leq \theta \leq 2\pi$$

$$1 \leq r \leq 5$$



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$R:$

$0 \leq \theta \leq 2\pi$

$1 \leq r \leq 5$

$$\int_R \int (x^2 + y) \, dA =$$

Recall: $\cos^2(\theta) = \dfrac{1 + \cos(2\theta)}{2}$

Double Integrals with Polar Coordinates!

As with rectangular coordinates, we can bound by functions as well...

If f is continuous on a polar region of the form

$$D = \{(r, \theta) \mid \alpha \leqslant \theta \leqslant \beta,\ h_1(\theta) \leqslant r \leqslant h_2(\theta)\}$$

then
$$\iint_D f(x, y)\, dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$